

FIGURE 66. CROSS-SECTIONAL VIEW OF HYDROSTATIC-EXTRUSION TOOLING

conservative side for predicting stresses that would produce yielding in shear. Therefore, it was decided to base the container design on the Hencky-Von Mises or maximum-distortion-energy criterion.

The Hencky-Von Mises theory holds that a material subjected to a three-dimensional stress system will yield when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 \sigma_y^2 = 6 K^2$$

where

 σ_1 , σ_2 , σ_3 = principal stresses

σ_y = yield stress as determined in uniaxial tensile or compressive tests

K = yield stress in pure shear.

In this case, for a container assembly, the stresses are considered to be biaxial because there is no axial load on the vessel. The hoop stresses are usually tensile and the radial stresses are always compressive. These two stresses will be the principal stresses because there are not externally applied shear stresses in the system. High resulting shear stresses can be expected when the system consists of two principal stresses of opposing sign.

Under biaxial conditions the Mises yield criterion becomes:

$$\sigma_1^2 - (\sigma_1 \sigma_3) + \sigma_3^2 = \sigma_y^2 = 3 \ \text{K}^2 \quad .$$

This equation predicts that yielding will occur when the stress in pure shear becomes equal to 0.577 Y. This value is equivalent to the maximum-shear-stress criterion provided that the yield stresses in pure tension or compression are multiplied by $2\sqrt{3}$. With that modification of the Tresca criterion, solutions determined by either relationship agree within approximately two to six percent.

Therefore, it was decided that the container would not be expected to deform plastically, and the design would be acceptable, if the stressed metal in the vessel met either of the following equivalent limiting conditions:

$$\frac{(\sigma_1^2 - \sigma_1 \ \sigma_3 + \sigma_3^2)^{0.5}}{\sqrt{3}}$$
 <0.577 Y

Modified Tresca

$$\frac{\sigma_1 - \sigma_3}{2}$$
 <0.577 Y

where

 σ_1 = hoop stress at the inside of the liner

 σ_3 = radial stress at the inside of the liner.